1. Two parallel, plane circular disks (of radius R) lie one above the other a small distance apart. The space between them is filled with a fluid. The upper disk approaches the lower at a constant (small) velocity U, displacing the fluid. The pressure at r = R is po and pr is not a function of z. The simplified forms of the basic equations (continuity and motion, r component) are given below.
2. Show how these simplifications are achieved using an analysis based on your understanding and estimate of the magnitudes of the different terms in the original equations.
3. Solve the these equations with proper boundary conditions to show that the resistance to motion on the moving disk is F = (3 µ U R 4)**/**(2 h 3 ), where h is the separation between the two plates. Take the origin to be at the centre of the lower (stationary) plate.

[Hint: Solve equation of motion first, then equation of continuity to obtain pressure as a function of r and other system parameters]

2. Consider the spin-coating process used to coat silicon wafers with photoresist. The process is designed to produce a very thin, uniform coating by spinning a viscous, Newtonian, liquid onto a substrate (wafer). The process has angular symmetry, the rotation rate is constant, and since the film is thin, there are no real pressure gradients or fluid accelerations and body forces to speak of. The thin film also moves with the substrate as if it were a rigid body, vθ ≠ ƒ(z).

a) Show that the continuity and momentum equations reduce to:

Coating

vz

vr

r

z

h

ω

Wafer

 

b) What are the boundary conditions for this problem?

c) Solve the equations for vr and vz.

d) The velocity, vz, at the film/air interface is just the change in film thickness with time. Use this to obtain a differential equation for h, and integrate this equation to obtain the solution as:

 (check for correctness)

Where ω is the rate of rotation (vθ = rω) and h0 is the initial height of the film.

Special Note: This is one of the possible solutions that was obtained by equating the depth average velocity in the z direction < vz > to the change in film thickness with time, dh/dt.